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Guidebook Contents

Parts 1, 2, and 3

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Course Guidebook

Algebra I

Taught by: Monica Neagoy
Georgetown University

Parts 1, 2, and 3

Course Guidebook

THE TEACHING COMPANY®

Monica Neagoy, Ph.D.
Georgetown University

Dr. Monica Neagoy has been a professional lecturer in Mathematics at Georgetown University since 1980. She essentially designs, directs, and teaches institutes for teachers and students of mathematics. In addition, Dr. Neagoy serves the community as a mathematics/mathematics education consultant on a wide variety of mathematical content and method areas. Countless private schools and public school systems are listed among her clients, as well as private organizations such as The Annenberg Foundation, Public Broadcasting Systems, The Cafritz Foundation, and The Inter- national Satellite Corporation (INTELSAT). Finally, she is highly esteemed in the mathematical community on both a national and international level: she is one of seven national judges for the prestigious MATHCOUNTS organization, and is frequently invited to speak—often to give the keynote address—at conferences and special mathematics events in the United States and abroad.

On another note, Dr. Neagoy co-directs the professional LE NEON French-American Theater in Arlington, Virginia where she does everything from acting and directing to fund raising and public relations. Also on the "artistic" side, she has made her first film, *Five Days in Paris*, which premiered in April 1994 at the Kennedy Center for the Performing Arts as a part of FILMIEST DC. Dr. Neagoy had the lead role in this film.

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